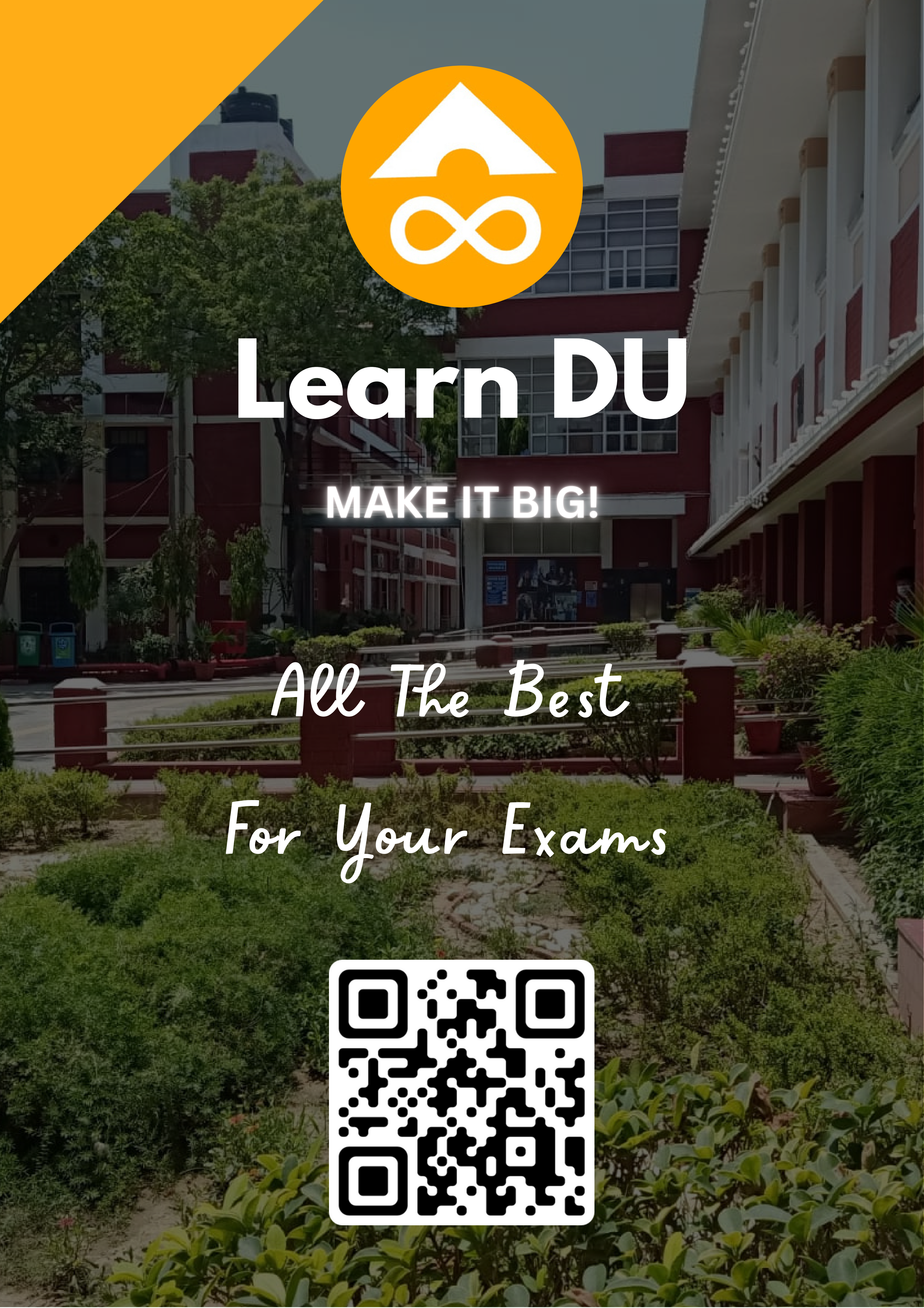
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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7483**

Unique Paper Code : **32221301** **J**

Name of the Paper : **Mathematical Physics-II**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Section-A

1. (*a*) Write a general expression for the Fourier series of a function $f(x)$, such that $f(x) = f(x + 2L)$, $-L < x < L$. Which terms will be missing if $f(x)$ is an even function ? Justify mathematically. 6

Or

Evaluate $\int_{-L}^{L} \cos \frac{p\pi x}{L} \cos \frac{q\pi x}{L} dx$ for : 6

(*i*) $p = q \neq 0$

(*ii*) $p \neq q$.

(b) Plot the periodic function defined by : 2,6,4

$$f(x) = -\pi, \quad -\pi < x < 0$$

$$f(x) = x, \quad 0 < x < \pi$$

Find the Fourier series of this function and hence prove that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$$

(c) What is the period of sin nx and that of tan x. 2

Or

If $f(t + T) = f(t)$, then show that :

$$\int_a^b f(t)dt = \int_{a+T}^{b+T} f(t)dt\,.$$

Section-B

2. (a) Classify the point $x = 0$ as a regular or irregular singular point for the differential equation : 3

$$x^2 \frac{d^2y}{d^2x} + \sin x \frac{dy}{dx} + e^{-x} y = 0\,.$$

(*b*) Solve the following differential equation about $x = 0$, using Frobenius method :

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0.$$

Or

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left[x^2 - \frac{1}{4}\right]y = 0$$

3. Attempt any *two* parts : 2×7.5=15

(*a*) Prove that :

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi}\cos(n\theta - x\sin\theta)d\theta,\ n = 0, 1, 2 \ldots$$

(*b*) Expand $f(x) = x^2 - 3x + 2$ in a series of the from $\sum_{k=0}^{\infty} A_k P_k(x)$, using $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{3x^2-1}{2}$.

(*c*) Using the generating function for Bessel's Polynomials or otherwise, prove that :

$$xJ'_n(x) = -nJ_n(x) + xJ_{n-1}(x)$$

(*d*) Obtain an expression for $P_4(x)$ using appropriate formula.

4. Attempt any *one* part :

(*a*) Evaluate :

$$\int_0^1 \frac{dx}{\sqrt{-\ln x}}$$

(*b*) Evaluate :

$$\int_0^a y^4 (a^2 - y^2)^{1/2} dy$$

(*c*) Prove that :

$$\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{1}{a^n b^m} \beta(m, n).$$

Section-D

5. (*a*) The solutions to 2-D wave equation are obtained as trigonometric functions as well as in terms of Bessel functions. Explain how trigonometric cosine function is different from the Bessel Function of Order Zero. Compare them in terms of :

(*i*) Periodicity

(*ii*) Amplitude

(*iii*) Zeros.

Indicate differences using a plot.

Or

Using the method of separation of variables, solve : 5

$$\frac{\partial u}{\partial y} = 2\frac{\partial^2 u}{\partial x^2}; \quad 0 < x < 3, \quad y > 0$$

Given $u(0, y) = u(3, y) = 0$, and $u(x, 0) = 5 \sin 4\pi x$ $-3 \sin 8 \pi x$.

(*b*) Find the steady state temperature, $u(x, y)$ of a rectangular plate $(0 < x < 1; 0 < y < 2)$ subject to the boundary conditions : $u(x, 0) = 0$, $u(0, y) = 0$, $u(1, y) = 0$, and $u(x, 2) = x$. 10

Or

Using the method of separation of variables, solve 1-D wave equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

Subject to conditions $y(0, t) = 0$, $y(L, t) = 0$ and

$$y(x,0) = \begin{cases} x, & 0 < x < \frac{L}{2} \\ L - x, & \frac{L}{2} \le x \le L \end{cases}, \quad y_t(x, 0) = 0$$

where $y_t = \frac{\partial y}{\partial t}$. 10

(c) Show that $u(x, t) = e^{-8t} \sin 2x$ is a solution to $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ with the conditions $u(0, t) = u(\pi, t) = 0$, $u(x, 0) = \sin 2x$.

Or

Using the method of separation of variables, prove that the general solution of $\frac{\partial f}{\partial t} = 4\frac{\partial f}{\partial x}$ is given by :

$$f(x,t) = Ae^{k\left[\left(\frac{x}{4}\right)+t\right]}$$

where A and k are some constants 5

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